**Taylor Series**

**In-class investigation Total marks: 48**

A Taylor series uses the sum of an infinite number of polynomial terms to represent a non- polynomial function. Even though such a sum cannot be calculated, an approximation of the function can be obtained by using a finite number of terms. The more terms used, the better the approximation.

To complete this task, you will need to have an understanding of

1. polynomial differentiation: if and
2. factorial notation:

**Question 1 (21 marks)**

Consider the Taylor series

1. The Taylor polynomial formed by the first 4 terms of is

Graph on the axes below. Write the coordinates of any intercepts and turning points onto your graph. (6)

**x**

-

5

5

**f(x)**

-

3

-

2

-

1

1

2

3

1. Continue the series for another 3 terms and hence write . (3)
2. Graph on the axes below. Write the coordinates of any intercept and turning points onto your graph. (6)

**x**

-

5

5

**f(x)**

-

3

-

2

-

1

1

2

3

1. Suggest the function represented by the Taylor series, (2)
2. Suggest values of *x* for which provides a good approximation of the function when
   1. four terms of the series are used. (2)
   2. seven terms of the series are used. (2)

**Question 2 (13 marks)**

Consider the Taylor series

1. Continue the series for another 3 terms and hence write . (3)
2. Graph on the axes below. Write the coordinates of any intercept and turning points onto your graph. (6)

**x**

-

5

5

**g(x)**

-

3

-

2

-

1

1

2

3

1. Suggest the function represented by the Taylor series, (2)
2. Suggest values of *x* for which provides a good approximation of the function when seven terms of the series are used. (2)

**Question 3 (10 marks)**

Recall the Taylor series from Questions 1 and 2

and

1. Show that . (4)
2. Determine and comment on the result. (4)
3. State the relationships from (a) and (b) above in terms of the functions you suggested for and in Questions 1(d) and 2(c). (2)

**Question 4 (4 marks)**

Another Taylor series is defined by

1. Determine . (2)
2. Suggest the function for which is a good approximation. (2)

**End of questions**